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O MRS-GARCH SUPERA OS MODELOS DO TIPO GARCH PARA PREVISÃO?

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RESUMO
Este trabalho avaliou se os modelos GARCH do regime de Markov podem superar os modelos do tipo GARCH na previsão. Para atingir este objetivo, foi utilizado o mercado acionário brasileiro - Ibovespa - entre 2012 e 2017 como escopo do estudo. A principal diferença do nosso trabalho é a definição do padrão dentro e fora da amostra. Este trabalho determinou uma previsão de 200 passos à frente (10 meses), com reestimação do modelo a cada passo à frente, a fim de encontrar evidências conclusivas e resultados robustos do modelo que tem melhor capacidade preditiva. Os resultados mostraram que os modelos do tipo GARCH mostram um desempenho ligeiramente melhor para o VaR a 5% e os modelos de regime de Markov tiveram melhor desempenho a 1% e precisão preditiva considerando a maioria dos critérios estatísticos. Além disso, conclui-se que nenhum modelo poderia ser determinado como referência por critérios estatísticos, o que mostra que não há como determinar um modelo que supera a previsão no mercado acionário brasileiro.

Palavras-chave: Previsão; Value-at-risk; Volatilidade; MRS-GARCH; Mercado acionário

DOES MRS-GARCH OUTPERFORMS THE SINGLE GARCH-TYPE MODELS FOR FORECASTING?

ABSTRACT
This paper evaluated if the Markov switching regime GARCH models can outperforms the single GARCH-type models on forecasting. To achieve this objective, it was used the Brazilian stock market – Ibovespa – between 2012 and 2017 as study scope. The major difference of our work is the definition of the in-sample and out-of-sample pattern. This paper determined a 200 steps-ahead (10 months) forecast, with model re-estimation at each step-ahead, in order to find conclusive evidence and robust results of the model which has better predictive ability. The results showed that the single GARCH-type models show a slightly better performance for VaR at 5% and switching regime models had better performance at 1% and predictive accuracy considering the most of statistical criteria. Besides that, no model could be determined as benchmark by statistical criteria, which displays that there´s no way to determine a model that outperforms for forecasting on the Brazilian stock market.

Keywords: Forecasting; Value-at-Risk; Volatility; MRS-GARCH; Stock market.

JEL: C53; C58.

1 INTRODUCTION

The volatility researches in financial series became popular from pioneering studies by Engle (1982) and Bollerslev (1986), which estimated ARCH

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(Autoregressive conditional heteroskedasticity) and GARCH (Generalized autoregressive conditional heteroskedasticity) models. The reason for these models became widely used was, according to Reher and Wilfling (2011), the compatibility of the GARCH models with the financial series of returns, the existence of efficient statistical methods to estimate the parameters of the model and availability of useful forecasts for volatility.

Besides that, measuring risk has become a crucial issue for many portfolio managers and investors. In recent years, finance literature has focused on risk management. So, Value-at-Risk (VaR) analysis has been a matter of great concern for financial risk management. VaR analysis has been extensively used to measure the possible maximum amount of loss for an asset (or portfolio) in a specific period of time at a given confidence level by portfolio managers, regulators and practitioners. In other words, VaR has measured the maximum loss in value of a portfolio over a predetermined time period for a given confidence level (BALIBEY; TURKYILMAZ, 2014).

In order to search for specific models for each type of volatility such as leverage and stylized facts, several GARCH-type model specifications have been developed. The EGARCH models of Nelson (1991) and GJR-GARCH of Glosten *et al*. (1993), for example, consider asymmetries and volatility shocks.

Although the EGARCH and GJR-GARCH models assume the possibility of volatility asymmetry, they operate on only one regime to capture the high persistence in volatility, ie, they adjust to the historical series with only one pattern, ignoring possible structure changes. Cai (1994) and Hamilton and Susmel (1994) introduce the regime change process (Hamilton, 1988, 1989) into the GARCH model in order to consider possible structural breaks. In particular, the Markov Regress Switching Model (MRS-GARCH) allows Markov chain regimes to have different GARCH behaviors, i.e, different volatility structures, to extend the fit of GARCH models to dynamic forms and to perform better estimations, forecasting and prediction performance (KLAASSEN, 2002; HAAS et al., 2004; MARCUCCI, 2005; ZHANG; WANG, 2015; ZHANG; ZHANG, 2015).

According to Reher and Wilfling (2011), the GARCH models with switching regimes are designed to capture discrete changes in the volatility process of the data series, and are widely used in time series. Some studies of this approach make use of commodity prices, such as those of Alizadeh et al (2008), Henry (2009), Bohl et al (2011) and Zhang, Yao & He (2015). Other works make use of series of returns of some listed companies, as is the case of Reher and Wilfling (2011) applied in the German market and Chlebus (2016), which analyzed the Polish market, specifically companies listed in the Warsaw Stock Exchange.

Besides that, some works focused on testing the accuracy for forecasting of MRS-GARCH against the single GARCH-type models. Liu and Hung (2010) proposed two types of regime-switching GARCH models and the single GARCH-type to forecasts the YEN-US Dollar exchange rate and IBM stock price and confirmed the better performance for the regime-switching models. Clifter (2013) forecasted the electricity prices of Nordic electric power market with different volatility models and conclude that the MRS-GARCH outperformed all other models of single regime for forecasts electricity prices. Iqbal (2016) forecasts the returns of KSE – that is the leading and dominant stock exchange of Pakistan – using the single GARCH-type models besides the MRS-GARCH. He concluded that the MRS-GARCH outperformed forecasting for short-time horizons (1 and 5 day ahead), and EGARCH with *t-student* distribution performed better for long-time horizons (22 day ahead).

When observing the difference between the single regime GARCH-type and those of Markov, this study aims to examine if the MRS-GARCH models improves the forecasts of volatility and VaR, compared to the single GARCH-type models. Gaussian, students-t and GED distributions are compared in terms of accuracy of volatility forecasts. The robustness of this work lies in the fact that we're using 200 steps ahead with model re-estimation at each one step.

2 ECONOMETRIC METHODS

This section will be structured in three as follows: first, the GARCH-type models will be showed. Then, the Markov Switching Regime GARCH-type will be exposed. Thus, the Value-at-risk method will be described. After that, the accuracy of prediction volatility models and his criterias, and for last, all the criterias of VaR backtesting will be showed.

2.1 GARCH-type models

Bollerslev (1986) defined the GARCH (*p,q*) model, which consider the current conditional volatility as function of *p* previous conditional variances *q* previous squared errors. This model can be expressed as (1).

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_1 \varepsilon_{t-1}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2; t \epsilon \mathbb{Z}. \quad (1)$$

Where $\alpha_0 > 0$, $\alpha_1, \beta_1 \geq 0$, and $\alpha_1 + \beta_1 < 1$, according to Iqbal (2016), ensure a positive conditional variance and stationarity of the process. This model is symmetric, that is, both negative and positive shocks have similar impact on the conditional volatility.

To capture asymmetries, there are two popular volatility models that can respond asymmetrically to positive and negative shocks. The exponential GARCH (EGARCH) model of Nelson (1991) and the GJR model of Glosten et al (1993). The EGARCH model can be expressed as (2).

$$\log(\sigma_t^2) = \alpha_0 + \sum_{i=1}^{p} \alpha_1 \frac{\varepsilon_{t-1}}{\sigma_{t-1}} + \alpha_2 \left(\left| \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \right| - E \left| \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \right| \right) + \sum_{t-1}^{q} \beta_j log\, (\sigma_{t-j}^2). \quad (2)$$

In case $\alpha_2 < 0$, according to Wennström (2014), corroborates the leverage effect. In addition, if the parameter α_1 is statistically significant and non-zero, there is an asymmetric effect. This effect is characterized by the difference of responses to shocks in the series, i.e, a positive shock does not have the same effect as a negative shock of the same magnitude. According to Alexander *et al*. (2009), the term β is the parameter that measures the persistence of volatility, and the higher the value, the more intense the volatility and its persistence lasts longer.

The other way to model the impact of asymmetric shock is the GJR-GARCH, that can be described as follows in (3).

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} [\alpha_1\, \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-1}^2 I_{(\varepsilon_{t-i>0})}] + \sum_{j=1}^{q} \beta_j\, \sigma_{t-j}^2. \quad (3)$$

Where $I_{(.)}$ represents a function, who presents a unitary value if $\varepsilon_{t-1} < 0$, $\forall_i = 1,2,\ldots,p$ and zero if $\varepsilon_{t-1} > 0$, $\forall_i = 1,2,\ldots,p$. Iqbal (2016) infers that the positive return contributes to the volatility only through the factor $\alpha_1 + \alpha_2$ in case of negative return. Here α_2 is called the asymmetric parameter, a significant $\alpha_2 > 0$ indicates leverage effect and the $\alpha_2 = 0$, the GJR (1,1) model reduces to the linear GARCH (1,1) model.

2.2 Markov regime-switching GARCH models

Using the definition of Kritzman et al (2012) and applied by Günay (2015), the Markov switching regimes are represented as follows in (4).

$$\Pr(X_i = i) = p_i, \tag{4}$$

where X_i is the first Markov regime. Next, will be showed the probability of switching regimes Γ , where $\gamma_{i,j}$ represents the parameters of each regime transition matrix.

$$\Gamma = \begin{pmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{pmatrix} \tag{5}$$

and

$$\gamma_{ij} = \Pr(X_i = j | X_{t-1} = i). \tag{6}$$

Where *t* shows time. So, the Markov chain will be on X_i = 1 or X_i = 2 regimes on time. Each regime presents γ_i observations and according the π_i distribution. This distribution can be explained as follows in (7).

$$\pi_i = \Pr(\gamma_i = s | X_i = i), \tag{7}$$

which demonstrates that the X_i regime dictates the probability γ_i have a *s* specific value. Following the GARCH-type models, the MRS-GARCH can be exposed as follows in (8).

$$r_i = \mu_1^{(i)} + \varepsilon_i = \delta^{(i)} + \varepsilon_i \ ; \ \acute{\eta}_i \sqrt{h_i}\ ; \tag{8}$$

$$h_1^{(i)} = \alpha_0^{(i)} + \alpha_1^{(i)} \varepsilon_{t-1}^2 + \beta_1^{(i)} E_{t-1}\left\{ h_{t-1}^{(i)} \middle| s_1 \right\}; \tag{9}$$

$$E_{t-1}\left\{ h_{t-1}^{(i)} \middle| s_1 \right\} = p_{ii,t-1}\left[\left(\mu_{t-1}^{(i)} \right)^2 + h_{t-1}^{(i)} \right] + p_{ji,t-1}\left[\left(\mu_{t-1}^{(j)} \right)^2 + h_{t-1}^{(i)} \right] - \left[p_{ii,t-1}\mu_{t-1}^{(i)} + p_{ji,t-1}\mu_{t-1}^{(i)} \right]^2. \tag{10}$$

Which i,j = 1,2 delimitates the two regimes of MRS-GARCH model, ^{ij} , $p_{ji,t} = \Pr(s_i = j | s_{t+1} = i, \varsigma_{t-1}) = \frac{p_{ji} \Pr(s_i = j | s_{t+1} = j, \varsigma_{t-1})}{\Pr(s_{t+1} = i | \varsigma_{t-1})} = \frac{p_{ji}\, p_{j,t}}{p_{i,t+1}}$, ς_{t-1} represents the information showed in $t-1$.

2.3 Value-at-Risk (VaR)

Method that measures the worst expected loss of a portfolio at a given confidence model, VaR follows the regulatory process of the Basel Accords (currently the Basel III Accords), banks and financial institutions are required to meet capital requirements, and must rely on state–of–the–art risk systems (ARDIA et al., 2016).

VaR is achieved in two steps. First, the distribution of future profit & loss (i.e., future portfolios or assets returns) is modelled. Second, financial risk is measured from the distribution; nowadays, the VaR risk measure is the standard (JORION, 1997). This metric gives, for a given time horizon, the asset's loss (or return) that is expected to be exceeded with a given probability level α (referred to as the risk level), and which is typically set to one or five percent, i.e, $\alpha \epsilon \{0.01, 0.05\}$. Hence, the VaR is nothing else than a given percentile of the returns distribution. The popularity of VaR mostly relies on: (i) the simple rationale behind it, (ii) the ease of computation, and (iii) its role in the financial regulation (BASEL COMITEE, 2010).

Hence, assuming a continuous cumulative density function (*c. d. f.*) with time-varying parameters $\theta_t \epsilon \mathbb{R}^d$ and additional static parameters $\psi \epsilon \mathbb{R}^d$, $F(.,\theta_t;\psi)$, for the asset log-return at time *t,* VaR is computed as follows in (11).

$$VaR_t \equiv F^{-1}(\alpha;\theta_t;\psi), \tag{11}$$

where $F^{-1}(.)$denotes the inverse *c. d. f.*, i.e, the quantile function. It follows that $VaR_t(\alpha)$ is nothing more than a α-quantile of return distribution at time *t*

2.4 Predictive Accuracy of volatility models

A huge number of statistical loss functions exists that can be evaluate the volatility forecasts models. According to Lopez (2011), there's no unique criteria for selection of the best model whose fits better for forecasting. This study employs two different statistical loss funcion to evaluate the forecasts of competing models. The root mean squared error (*RMSE*) and the mean percentage error (*MPE*) are commonly used for the evaluation of volatility forecasts. RMSE is defined as follows in (12).

$$RMSE = \sqrt{\frac{1}{T} >_{t=1}^{T} (\sigma_t - \hat{h}_t^{\frac{1}{2}})^2}. \tag{12}$$

An RMSE with proximity of zero means that the quality of prediction gets closer of perfection. MPE is defined by (13).

$$MPE = \frac{100\%}{n} \sum_{t=1}^{n} \frac{a_t - f_t}{a_t}. \tag{13}$$

Where $a_t$is the actual value of the quantity being forecast, f_t is the forecast, and *n* is the number of different times for which the variable is forecast. As occurs on *RMSE*, values that getting closer from zero are preferred for *MPE* criteria.

2.5 VaR backtesting

To test a VaR estimator, Kupiec (1995) developed a test based on the number of failures in the market risk forecast, compared to the confidence interval established before. The test is based on the likelihood ratio, which can be used to estimate a point sample statistically consistent with the VaR model. This statistic follows a chi-square distribution with 1 degree of freedom, and is given by (14).

$$LR_{uc} = 2ln[(1-\hat{\alpha})^{T-T}\hat{\alpha}^{T\cdot}] - ln[(1-\alpha)^{T-T}\alpha^{T\cdot}] \sim \chi^2. \quad (14)$$

This test only verifies the ponctual estimate of *VaR*. According to Coster (2013), the test proposed by Christoffersen (1998) aims to test whether the VaR is being punctually well estimated (that is, if the ratio of violations is close to the expected level) and whether these violations are independent. The Christoffersen test is divided into two parts, one, called by the author as unconditional coverage (LR_{uc}), which is used to verify the ponctual estimate and another, called the independence test (IT), to verify the independence of such violations. However, the author emphasizes thet alone is not able to test the independence of violations. The *IT* statistic can be obtained by (15) and (16).

$$IT = 2\left[(\ln(1-\hat{\pi}_{01})^{T_{00}}\hat{\pi}_{01}^{T_{01}}(1-\hat{\pi}_{11})^{T_{10}}\hat{\pi}_{11}^{T_{11}}) - \ln((1-\hat{\pi})^{(T_{00}+T_{10})}\hat{\pi}^{(T_{01}+T_{11})})\right], \quad (15)$$

$$\hat{\pi}_{ij} = {T_{ij}}\big/{(T_{i0} + T_{i1})}, \hat{\pi} = {(T_{01} + T_{11})}\big/{T}. \quad (16)$$

For i, j = 0,1, let T_{ij} denotes the number of time points $\{t; 2 \leq t \leq T\}$ for which $I_t = i$ is fallowed by $I_{t+1} = j$. Both tests are likelihood ratio tests and the final test statistic is the sum of these two test statistics. Thus, the conditional coverage test (LR_{cc}) is defined as (17).

$$LR_{cc} = TI + LR_{uc} \sim \chi_2^2. \quad (17)$$

Thus, the Christoffersen test has as null hypothesis that the proportion of VaR violations that occur is equal to the number of expected proportions and that these violations occur independently.

In addition to these tests, we have the Dynamic Quartile Test (*DQ*), which also measures the independence of VaR returns violations. According to Chen *et al.* (2011), it employs a regression-based model of the violation-related variable “hits”, defined as $I(y_t < -VaR_t) - \alpha$, which will on average be α if uncondicional coverage is correct. A regression-type test is then employed to examine whether the “hits” are related to lagged “hits”, lagged VaR forecasts, or other relevant regressors, over time.

To estimate the magnitude of the return violation, McAleer and Da Veiga (2008) proposed the absolute deviation (*AD*), which is defined by (18).

$$AD_t = |y_t - (-VaR_t)|I(y_t < -VaR_t). \quad (18)$$

Such as defined by Chen *et al.* (2011), this measure is related to the size of the loss for violating returns. The evaluated forecast of this study was based on the mean and maximum of *AD*, where the smaller values are preferred.

In order to evaluate and compare various VaR forecast models in terms of predictive quantile loss (*QL*), we use the “check function” of Gozález-Rivera et al. (2004). In what follows, the expected loss is given by (19).

$$QL = E\{\alpha - 1(y_t < q_t(\alpha))[y_t - q_t(\alpha)]\}. \quad (19)$$

The loss QL can provide a measure of the lack-of-fit of quantile model. The best forecasting model is the one that minimizes the expected loss. The null hypothesis is thet all the models are no better that eachother. Evidently, QL is an asymmetric loss function that penalizes more heavily with weight (1 − α) the observations for which we observe returns VaR exceedance. Quantile losses are then averaged over the forecasting period. Models with lower averages are preferred.

The referred tests presents the statistical validity of *VaR*. To test the accuracy of the model, we have the actual over expected excedence ratio (*AE*), which is defined as follows in (20)

$$AE = {}^{\hat{\alpha}}/_{\alpha}. \quad (20)$$

For being a relation between an estimated statistic ($\hat{\alpha}$) and realized (α), values close to 1 will be preferred, whereas they present a rate closer to that achieved.

3 DATA AND EMPIRICAL RESULTS

In order to verify what the model that outperforms on forecasting, this study was focused on the Brazilian stock market, more specifically on Ibovespa. The Bovespa Index or Ibovespa (Ibov) is the main indicator of the Brazilian stock market that validates the average performance of stock prices traded on the Sao Paulo Stock Exchange. It is formed by the stocks with the highest volume traded in recent months. Ibovespa's objective is to be the indicator of the average performance of the quotations of the assets of greater negotiability and representativeness of the

Brazilian stock market (BM&F BOVESPA 2018). The quotation values of the index was collected from January 4, 2012 until December 12, 2017 with daily frequency, which totalized a universe of 1.502 observations.

The return at time t is defined as r_t = [log(p_t) – log(p_{t-1})], for t = $1, \ldots, T$, where $p_t$is the closing if Ibovespa at time t. Initially, all models were estimated using sample of 1301 observations, what we called in-sample. For volatility and VaR forecasting, a rolling sample scheme is used to forecast the day-ahead, and the models are fitted using the original samples of the 200 steps initially omitted, fitting one by one until you reach the two hundredth day of forecast, i.e, the sample is rolled forward and the models are re-fitted using the original samples (1401 + $i, for\ i$ = $1, \ldots,$200) of the 100 steps (or 10 months) initially omitted The 200 observations ahead we called out-of-sample.

Either Ibovespa than return of Ibovespa are shown at time Figure 1. It can be noticed that the sharpest fall by all time horizon occurred at 2016, a year of much political turbulence at Brazil, what can have some impact on the economy and the stock market. Observing the returns, it can be noticed some peaks, both high and low in many parts of the series, which demonstrates some volatility on the series.

Summary descriptive statistics for the series are shown at Table 1. The negative value of mean shows that the Brazilian market suffered with some retraction between the analyzed periods. The standard-deviation shows a slightly volatility on the Brazilian stock market. The kurtosis follows a platykurtic line. The skewness indicates that the series have a position at the right of the normal distribution. Both indicates that the series does not have a normal distribution behavior, besides a “fat-tailed” distribution, which is a stylized fact in financial series.

Table 1 – Descriptive statistics of Log-returns of Ibovespa, daily, between 2012-2017

Statistics	**Ibovespa's log-return**
Mean	-0.020
Median	0.008*
Minimum	-0.083
Maximum	0.101
Standard deviation	0.015
Kurtosis	2.396
Skewness	0.026

* Values multiplicated for 100
Source - Authors

First, GARCH-type (GARCH, EGARCH and GJR) and the MRS-GARCH models are fitted using Normal, Student t and GED distributions, for in-sample. The AIC criteria was used to determinate the distributions. The results of the best models selected by LL and AIC criteria are showed at the Table 2.

Table 2 – Estimation of GARCH-type and MRS-GARCH models (in-sample) with daily bases between 2012-2017

Ibovespa	GARCH (*st)*		GJRGARCH (*st)*		MRS-GJRGARCH (*st)*	
	Estimate	p-value	Estimate	p-value	Estimate	p-value
$\alpha_0^{(1)}$	0.000	0.000	0.000	0.000	0.000	0.000
$\alpha_1^{(1)}$	0.049	0.000	0.009	0.000	0.000	0.000
$\alpha_2^{(1)}$	-	-	0.073	0.001	0.000	0.000
$\beta^{(1)}$	0.924	0.000	0.935	0.000	0.345	0.000
$\upsilon_1{}^{(1)}$	14.326	0.000	14.737	0.160	5.214	0.000
$\upsilon_2{}^{(1)}$	1.052	0.000	1.047	0.001	0.780	0.000
$\alpha_0^{(2)}$	-	-	-	-	0.000	0.000
$\alpha_1^{(2)}$	-	-	-	-	0.008	0.000
$\alpha_2^{(2)}$	-	-	-	-	0.075	0.000
$\beta^{(2)}$	-	-	-	-	0.940	0.000
$\upsilon_1{}^{(2)}$	-	-	-	-	18.020	0.000
$\upsilon_2{}^{(2)}$	-	-	-	-	1.097	0.000
$Persistence^{(1)}$	-	-	-	-	0.998	0.000
$Persistence^{(2)}$	-	-	-	-	0.999	0.000
$Regime^{(1)}$	-	-	-	-	0.241	-
$Regime^{(2)}$	-	-	-	-	0.759	-
LL:	3600.964		3607.669		3610.496	
AIC:	-7191.928		-7203.338		-7192.992	

(s*t*) skewed t-student distribution; $^{(1)}$: First Regime; $^{(2)}$: Second regime
Source – Authors

Analyzing the GARCH-type models, it can be noticed that all the parameters of the models achieved statistical significance at 1%. Assessing the GJR model, despite the asymmetry parameter is statistically significant, there´s no presence of leverage effect. All the single regime models showed a high persistence of volatility in their estimates.

Looking the MRS-GARCH, as happens in the single models, all the parameters achieved statistical significance at 1%. The model shows either leverage effect and asymmetry only at the second regime by the parameter α_2 > 0, besides a behavior of volatility impacted by past shocks, by the parameter α_1 > 0, being this characteristic stronger at second regime of the series. Considering the persistence, the second regime stands out first because β is extremely higher. It can be noticed that the first regime of Ibovespa is present less than the second, what means that the more volatile, persistent and asymmetric regime figures longer.

Considering the regime persistence, the second performed the most part of the time, what means that this is the more apparent for the series. In relation of volatility, both regimes showed great persistence, what indicates a volatility with long memory.

Finally, with the best models estimated VaR at 5% and 1% forecasting is evaluated for the out-of-sample observations. The Table 3 shows the results of backtesting and predictive accuracy of each model estimated, 200 steps ahead.

Table 3 – Backtesting and predictive accuracy for VaR (out-of-sample) with daily bases between 2012-2017

Ibovespa	GARCH (*st*)		GJRGARCH (*st*)		MRS-GJRGARCH (*st*)	
	VaR		VaR		VaR	
	5%	1%	5%	1%	5%	1%
Backtesting for VaR						
LRuc	1.954	0.619	1.054	0.619	1.954	0.000
p-value	0.162	0.432	0.305	0.432	0.162	1.000
LRcc	2.327	0.629	1.564	0.629	2.327	0.041
p-value	0.312	0.730	0.457	0.730	0.312	0.980
AE	0.600	0.500	0.700*	0.500	0.600	1.000*
ADmean	0.017	0.070	0.014*	0.070	0.017	0.034*
ADmax	0.080	0.070	0.080	0.070	0.078*	0.066*
DQ	4.727	0.559	3.469	0.558	2.118	1.900
p-value	0.579	0.997	0.748	0.997	0.909	0.929
Loss (x100)	0.164	0.069	0.167	0.070	0.166	0.072
RMSE (x100)	0.558	0.491	0.559	0.492	0.546*	0.461*
MPE	7.489	2.132	7.442	2.141	7.327*	1.996*

Out-of-sample evaluation of 200 1-day-ahead VaR predictions (200 steps forward, with re-estimation the model at each 1 step); * Indicates the model with the better statistic parameter considering at least 5%.
Source - Authors

Analyzing the backtesting of VaR at 5%, it can be noticed that single GARCH-type model represented by GJR-GARCH two criteria against only one of MRS-GARCH, considering only the tests that have p-value at least of 5%. However, when evaluating the DQ criteria, which indicates a benchmark, no model outperformed other by the level of 5%, i.e, statistically no model proved to be superior.

However, with VaR at 1%, MRS-GARCH showed a slightly superiority against the other single GARCH-type models. The regime switching outperformed in 3 criteria at 5% against any of the single. As occur for VaR at 5%, evaluating the DQ criteria, no model outperformed other, which indicates that no model proved to be statistically superior than the other.

Now, when we analyze the predictive accuracy it can be noticed that the MRS-GARCH outperformed the single GARCH-type models in all the criteria, which indicates that the Markov switching regimes model performed a more closely to perfection forecast compared to other.

These results find support on the literature. Zhang, Yao & He (2015) study shows that the MRS-GARCH was accurate to forecast daily returns, but loses his superiority on weekly and month returns. Iqbal (2016) finds report that the single GARCH-type models are more accurate to forecast at long horizon (22 day-ahead) and the MRS-GARCH at short horizon (1 and 5 day-ahead).

Despite there's no benchmark model, switching regime model seems to be slightly better considering most of the statistical criteria. Yet, because there weren't statistically robust results, there is no way to say that the switching regimes GARCH models outperforms the single, even evaluating 200 steps-ahead forecasting, which theoretically could benefit a model that operates in multiple equations as occur in MRS-GARCH.

4 CONCLUSION

This paper evaluated if the Markov switching regime GARCH models can outperforms the single GARCH-type models on forecasting. To achieve this objective, it was used several statistical criteria to evaluate which model had more solid VaR and predictive accuracy, using the Brazilian stock market – Ibovespa – between 2012 and 2017 as study scope.

First, we defined the in-sample and out-of-sample pattern and determine a 200 steps-ahead (10 months) forecast, with model re-estimation at each step-ahead. Then, using AIC and Logarithm of Likehood criteria, GARCH-st, GJR-GARCH-st and MRS-GJR-GARCH-st models were elected the better on in-sample evaluation.

The out-of-sample forecasts showed that the single GARCH-type models had a slightly better performance on VaR at 5% and the switching regime models had a slightly better performance on VaR at 1%. The MRS-GARCH performed better on the predictive accuracy considering the most of statistical criteria. However, the DQ criteria – which delimitates the benchmark model – showed that no model outperformed other, which indicates that it can’t be determined a better model for forecasting considering the Brazilian stock market.

Other studies found no consensus on the ability to forecasts of the several GARCH-type models, being that the switching regime models performed better in some situations and the single regime models in other, which demonstrates that the results of this paper find support on the literature.

The main limitations of this work were the lack of comparison on other stock markets, which could show some difference on regimes and supports a model as superior than other for forecasts. The main suggestion for future research is to compare several indexes, specially those that have very different regimes, which can show if the switching regimes model beats the single regime for forecasting.

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